Comparison and Analysis of Channel Estimation Techniques in performance for Wireless OFDM System

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Abstract— Current and future communication schemes tend to use OFDM systems in order to provide high data transmission and less inter symbol interference.In this paper, we present the benefits of exploiting the *a* prior information about the structure of the Wireless channel on the performance of channel estimation for orthogonal frequency-division multiplexing (OFDM) systems. The work presented here mainly focus on channel Estimation for the various algorithm techniques which is used for OFDM system. The analytic treatment is complemented by thorough numerical investigation in order to validate the performance of the different techniques. A complete model including in both the time domain and the frequency domain is used for the multicarrier system, which models the received signal to noise ratio (SNR) of each subcarrier and further presents the estimation techniques which is describe the behavior of the OFDM system model in the wireless channel. On the other hand, the fast-varying fading amplitudes are tracked by using least-squares techniques that exploit temporal correlation of the fading process (modal filtering).

Index terms— Orthogonal Frequency Division Multiplexing (OFDM); Inverse Fast Fourier Transform (IFFT); Fast Fourier Transform (FFT); Cyclic Prefix (CP); Bit Error Rate (BER); Signal to Noise Ratio (SNR).

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1. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) could be tracked to 1950's, but it had become very popular at these days, allowing high speeds at wireless communications. OFDM could be considered either a modulation or multiplexing technique, and its hierarchy corresponds to the physical and medium access layer. A basic OFDM system consists of a QAM or PSK modulator/demodulator, a serial to parallel / parallel to serial converter, and an IFFT/FFT module. The combination of orthogonal frequency-division multiplexing (OFDM) and multiple-input multiple-output (MIMO) technologies, referred to as MIMO-OFDM, is currently under study as one of the most promising candidate for next-generation communications systems ranging from wireless LAN to broadband access. Recent works tackled the Performance assessment (both simulation and measurements) ofMIMO-OFDM systems in the presence of practical impairments, such as synchronization and channel estimation error [1]. As shown by references, channel estimation is a critical issue for MIMO-OFDM systems, especially if multilevel

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modulation is employed in order to achieve high spectral efficiencies. Orthogonal Frequency Division Multiplexing (OFDM) is a multicarrier transmission technique, which divides the available spectrum into many orthogonal carriers, each one being modulated by a low rate data stream. The low bit rate signals hardly suffer from inter symbol interference (ISI) in frequency selective channels, and because of orthogonality of the sub-carriers, it is possible to demodulate the received signal without crosstalk between the information on the subcarriers [6]. The effect of ISI on the OFDM system signal can be further improved by the addition of guard period to the start of each symbol. This guard period is a cyclic copy that extended the length of the symbol waveform. This guard interval is referred as cyclic prefix (CP). However, this cyclic prefix insertion can decrease the bandwidth efficiency greatly.

2. OFDM SYSTEM MODEL

In OFDM systems, data is transmitted on narrow-band subcarriers in frequency domain. Figure 2 shows some of these subcarriers in frequency domain. Sub-carriers have overlap in frequency domain, hence frequency efficiency is increased. If subcarriers are completely orthogonal, inter-channel interference (ICI) can be removed [3]. In this system, after pilot insertion between data sequence at the transmitter, the result data is modulated by inverse discrete Fourier transform (IDFT) on N parallel subcarriers and then after receiving signal at receiver transformed back to frequency domain by DFT. IN fact,
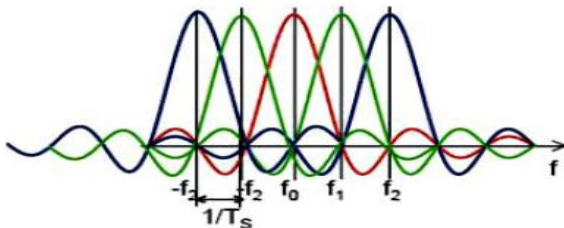
Fig 1. Sub-carrier in an OFDM system[3]

IDFT converts frequency domain data into time domain. The number of points of the IDFT/DFT is equal to the total number of sub-carriers. Every subcarrier can be formulatedas follow:

$$S_c(t) = A_c(t)e^{j[\omega_c t + \Phi_c(t)]} \qquad (1)$$

Where, A_c (t) is amplitude and ϕ_c (t) is phase. An OFDM signal is constructed from some of these subcarriers, so it can be described as follow:[3]

$$S_s(t) = \frac{1}{N}\sum_{n=0}^{N-1} A_n\ (t)e^{j[\omega_n t + \Phi_n(t)]} \qquad (2)$$

$$\omega_n = \omega_0 + n\Delta\omega$$

A_c (t) and ϕ_c (t) get different values in different symbols, but they are constant in every symbol and only depend on frequency of carriers. It means that we have in every symbol:

$$\Phi_n(t) \Rightarrow \Phi_n, A_n(t) \Rightarrow A_n \qquad (3)$$

If signal is sampled with $1/T$ (T is duration of a symbol) and refer"(3),"is inserted into "(2)," we will have,

$$S_s(kT) = \frac{1}{N}\sum_{n=0}^{N-1} A_n e^{j[\omega_n + n\Delta\omega)kT + \Phi_n]} \qquad (4)$$

It is obvious that with ω_0=0, refer (3) is converted to an IDFT transform. Therefore OFDM modulation is an IDFT transform inherently. In continuation cyclic prefix is inserted. Cyclic prefix is a crucial feature of OFDM that is used to prevent the inter-symbol interference (ISI) and inter-channel interference (ICI). ISI and ICI are produced by the multi-path channel through which the signal in propagated. Cyclic prefix protects orthogonality between sub-channels. The duration of the cyclic prefix should be longer than the maximum delay spread of the multi-path environment. For adding cyclic prefix, a part of the end of the OFDM time-domain waveform is added to the front of it. Cyclic prefix is caused that circular convolution is converted to linear convolution. Therefore the effect the channel on each subcarrier can be presented by a single complex multiplier please refer "(6),":

$$x_T(n) = \begin{cases} x(N+n), n = -N_{cp,} - N_{cp} + 1, \ldots -1 \\ x(n), = 0, 1, \ldots, N-1 \end{cases} \qquad (5)$$

$$Y(k) = S(k)H(k) + w(k) \qquad (6)$$

Where, $H(k)$ is the Fourier transform of channel impulse response (CIR). The frequency selective channel is modelled as a finite impulse response (FIR) filter.

$$h(n) = \sum_{i=0}^{L} g_i \delta(n - \lambda_i) \qquad (7)$$

is number of path and i, g is the channel gain in i^{th} path and is independent complex Gaussian random process with zero mean and unit variance, and λ_i is the delay of the ith path. Therefore [3],

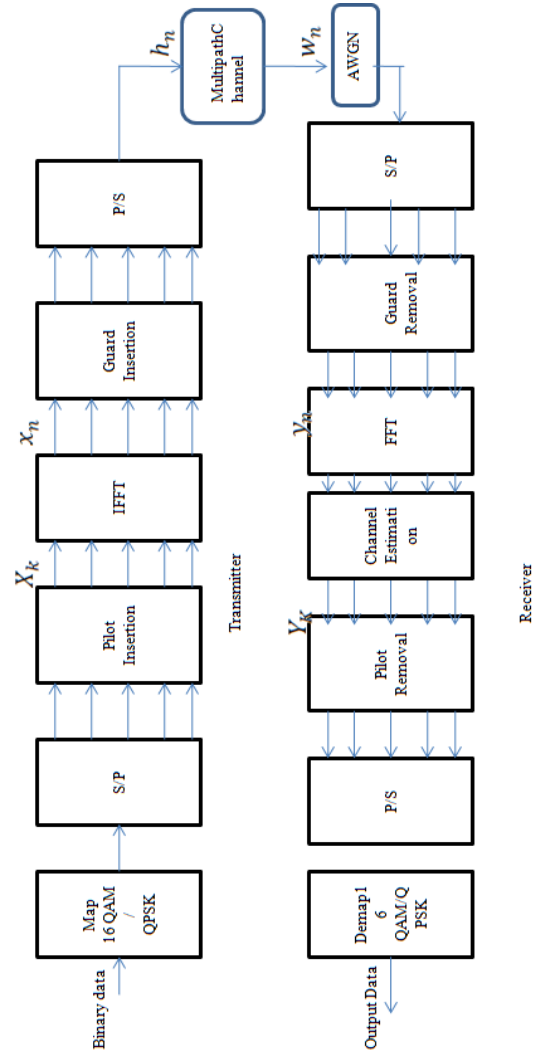$$H(k) = FFT\{h(n)\} = \frac{1}{N}\sum_{n=0}^{N-1} h(n)e^{-j(2k\pi n/N)} \qquad (8)$$

Figure 1 block diagram of an OFDM system

$K=0,1,\ldots,N-1$ Transmitted data, after passing through the channel and adding noise, is received as follow:

$$y(n) = s(n)\otimes h(n) + \omega(n) \qquad (9)$$

Where, y is received signal, s is transmitted data, and ω is additive white Gaussian noise. As shown in "Equation (10) received signal after removing cyclic prefix and applying FFT on it."

$$Y(k) = S(k)H(k) + W(k) \qquad (10)$$

That W and H are the Fourier transform of the noise and h respectively. In continuation, the channel is estimated in pilot subcarriers, and then whole channel frequency response is obtained by interpolation. And finally, data is detected as follow:[3]

$$X(k) = \frac{Y(k)}{H(k)} \qquad (11)$$

Where, $Y(k)$ is Fourier transform of $y(n)$. $X(k)$ and $H(k)$ are transmitted data and estimated channel respectively.

3. CHANNEL ESTIMATION

In an OFDM system, the transmitter modulates the message bit sequence into PSK/QAM symbols, performs IFFT on the symbols to convert them into time-domain signals, and sends them out through a (wireless) channel.

The received signal is usually distorted by the channel characteristics. In order to recover the transmitted bits, the channel effect must be estimated and compensated in the receiver. Each subcarrier can be regarded as an independent channel, as long as no ICI occurs, and thus preserving the orthogonality among subcarriers. The orthogonality allows each subcarrier component of the received signal to be expressed as the product of the transmitted signal and channel frequency response at the subcarrier. Thus, the transmitted signal can be recovered by estimating the channel response just at each subcarrier. In general, the channel can be estimated by using a preamble or pilot symbols known to both transmitter and receiver, which employ various interpolation techniques to estimate the channel response of the subcarriers between pilot tones. In general, data signal as well as training signal, or both, can be used for channel estimation. In order to choose the channel estimation technique for the OFDM system under consideration, many different aspects of implementations, including the required performance, computational complexity and time-variation of the channel must be taken into account. In the estimation procedure, The LS method is used to estimate the transmitted data symbols. These symbols will then be corrected by equalization procedure. Without using any knowledge of the statistics of the channels, the LS estimators are calculated with very low complexity, but they suffer from a high mean square error. Suppose, S_p is pilot signal matrix, H is specified channel condition matrix and R_p is received signal matrix, for pilot based channel estimation

$$H_{pLS} = [(S_p/R_p)]^T$$

MMSE can perform better in case of low SNR conditions but because of its high complexity it is not selected here. The performance depends upon the number of iterations with LS estimation.

3.1 TRAINING SYMBOL-BASED ESTIMATION

Training symbols can be used for channel estimation, usually providing a good performance. However, their transmission efficiencies are reduce due to the required overhead of training symbols such as preamble or pilot tones that are transmitted in addition to data symbols. The least-square (LS) and minimum-mean-square-error (MMSE) techniques are widely used for channel estimation when training symbols are available. We assume that all sub-carriers are orthogonal. Then, the Training symbols for N sub-carriers can be represented by the following diagonal matrix:

$$X = \begin{bmatrix} X[0] & 0 & \cdots & 0 \\ 0 & X[1] & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & X[N-1] \end{bmatrix} \qquad (12)$$

Where $X[k]$ denotes a pilot tone at the Kth sub-carrier, with $E\{X[k]\} =0$ and $Var\{X[k]\}=\sigma_x^2$ $k=0,1,2,\ldots,N-1$. Note that X is given by a diagonal matrix, since we assume that all sub-carriers are orthogonal. Given that the channel gain is $H[k]$ for each subcarrier k, the received training signal $\{Y[k]\}$ can be presented as per "(13)," Where H is a channel vector given as $H=[H[0], H[1],\ldots,H[N-1]]^T$ and Z is a noise vector given $Z=[Z[0],Z[1],\ldots,Z[N-1]]^T$ with $E\{Z[k]\}=0$ and $Var\{Z[k]\}=\sigma_Z^2$, $k=0,1,2,\ldots,N-$. In the following discussion, let $\hat{H}$ denote the estimate of channel H.

$$Y = \triangleq \begin{bmatrix} Y[0] \\ Y[1] \\ \vdots \\ Y[N-1] \end{bmatrix} = \begin{bmatrix} X[0] & 0 & \cdots & 0 \\ 0 & X[1] & \ddots & \vdots \\ \vdots & & & 0 \\ 0 & \cdots & 0 & X[N-1] \end{bmatrix} \begin{bmatrix} H[0] \\ H[1] \\ \vdots \\ H[N-1] \end{bmatrix} + \begin{bmatrix} Z[0] \\ Z[1] \\ \vdots \\ Z[N-1] \end{bmatrix} (13)$$

3.1 LS CHANNEL ESTIMATION

The least-square (LS) channel estimation method finds the channel estimate $\hat{H}$ in such a way that the following cost function is minimized:

$$J(\hat{H}) = \|Y - X\hat{H}\|^2$$
$$= (Y - X\hat{H})^H (Y - X\hat{H}) \qquad (14)$$

By setting the derivative of the function with respect to $\hat{H}$ to zero,

$$\frac{\partial J(\hat{H})}{\partial \hat{H}} = -2(X^H Y)^* + 2(X^H X\hat{H})^* = 0 \qquad (15)$$

We have $X^H X \hat{H} = X^H Y$, which gives the solution to the LS channel estimation as

$$\hat{H}_{LS} = (X^H X)^{-1} X^H Y = X^{-1} \qquad (16)$$

Let us denote each component of the LS channel estimate $\hat{H}_{LS}$ by $\hat{H}_{LS}[k], k=0, 1, 2,\ldots, N-1$. Since X is assumed to be diagonal due to the ICI-free condition, the LS channel estimates $\hat{H}_{LS}$ can be written for each sub-carrier as [4]

$$\hat{H}_{LS}[k] = \frac{Y[k]}{X[k]}, k = 0,1,2,\ldots,N-1 \qquad (17)$$

The mean-square error (MSE) of this LS channel estimate is given as [4]

$$MSE_{LS} = E\left\{(H - \hat{H}_{LS})^H (H - H_{LS})\right\} \qquad (18)$$

Note that the MSE in "(18)," is inversely proportional to the SNR, σ_x^2/σ_Z^2 which implies that it may be subject to noise enhancement, especially when the channel is in a deep null. Fig 4 shows the bit error rate for the BPSK modulation technique using LS algorithm

3.2 MMSE CHANNEL ESTIMATION

Consider the LS solution as per "(17)," $\hat{H}_{LS}=X^{-1}Y \triangleq \hat{H}$ Using the weight matrix W, define, $\hat{H} \triangleq W\tilde{H}$ which correspond to the MMSE estimate. Referring to below figure, MSE of the channel estimate $\hat{H}$ is given as

$$J(\hat{H}) = E\{\|e\|^2\} = E\left\{\|H - \hat{H}\|^2\right\} \qquad (19)$$

Then, the MMSE channel estimation method finds a better (linear) estimate in terms of W in such a way that the MSE in "(19)," is minimized. The orthogonality principle state that the estimation

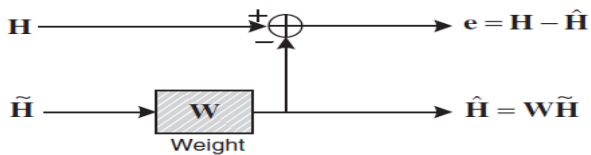Fig 2. MMSE channel estimation [9]

Errorvector $e = H - \hat{H}$ is orthogonal to $\tilde{H}$, such that [9]

$$= E\{H\tilde{H}^H\} - WE\{\tilde{H}\tilde{H}^H\} \qquad (20)$$

Where R_{AB} is the cross-correlation matrix of N×N matrices A and B (i.e., $R_{AB} = E[AB^H]$), and $\tilde{H}$ is the LS channel estimate given as

$$\tilde{H} = X^{-1}Y = H + X^{-1}Z \qquad (21)$$

Solving "(20)," for W yield

$$W = R_{H\tilde{H}}R_{\tilde{H}\tilde{H}}^{-1} \qquad (22)$$

Where $R_{\tilde{H}\tilde{H}}$ is the autocorrelation matrix of $\tilde{H}$ given as

$$= E\{HH^H + X^{-1}ZH^H + HZ^H(X^{-1})^H + X^{-1}ZZ^H(X^{-1})^H\} \quad (23)$$

$R_{H\tilde{H}}$ is the cross-correlation matrix between the true channel vector and temporary channel estimate vector in the frequency domain. "Using Equation (23), the MMSE channel estimate follows as" [9]

$$= R_{H\tilde{H}}\left(R_{HH} + \frac{\sigma_z^2}{\sigma_x^2}I\right)^{-1}\tilde{H} \qquad (24)$$

The elements of $R_{H\tilde{H}}$ and R_{HH} in "(24)," are

$$E\left\{h_{k,l}\tilde{h}_{k',l'}^*\right\} = E\left\{h_{k,l}h_{k',l'}^*\right\} = r_f[k - k']r_t[l - l'] \quad (25)$$

Where k and l denote the subcarrier (frequency) index and OFDM symbol (time) index, respectively. In an exponentially-decreasing multipath PDP (Power Delay Profile), the frequency-domain correlation $r_f[k]$ is given as

$$r_f[k] = \frac{1}{1 + j2\pi\tau_{rms}k\Delta f} \qquad (26)$$

Where $\Delta f = \frac{1}{T_{sub}}$ Is the sub-carrier spacing for the *FFT* interval length of T_{sub}. Meanwhile, for a fading channel with the maximum Doppler frequency f_{max} and Jake's spectrum, the time-domain correlation $r_t[l]$ is given as

$$r_t[l] = J_0(2\pi f_{max}lT_{sym}) \qquad (27)$$

Where $T_{sym} = T_{sub} + T_G$ for guard interval time of T_G and $J_0(x)$ is the first kind of 0th order [4] Bessel function. Note that $r_t[0] = J_0(0) = 1$, implying that the time-domain correlation for the same OFDM symbol is unity. Fig 5 shows the signal to noise ratio for BPSK modulation technique using MMSE algorithm.

3.4 DFT-Based Channel Estimation

The DFT-based channel estimation technique has been derived to improve the performance of *LS* or *MMSE* channel estimation by eliminating the effect of noise outside the maximum channel delay.

Let $\hat{H}[k]$ denote the estimate of channel gain at the kth sub-carrier, obtained by either *LS* or *MMSE* channel estimation method. Taking the IDFT of the channel estimate $\{\hat{H}[k]\}_{k=0}^{N-1}$

$$IDFT\{\hat{H}[k]\} = h[n] + z[n] \simeq \hat{h}[n], n = 0,1,\ldots,N-1 \qquad (28)$$

Where $z[n]$ denotes the noise component in time domain. Ignoring the coefficients $\{\hat{h}[n]\}$ that contain the noise only,

define the coefficients for the maximum channel delay L as

$$\hat{h}_{DFT}[n] = \begin{cases} h[n] + z[n], n = 0,1,2,\ldots,L-1 \\ 0, \qquad\qquad Otherwise \end{cases} \quad (29)$$

And transform the remaining L elements back to the frequency domain as follows

$$\hat{H}_{DFT}[k] = DFT\{\hat{h}_{DFT}(n)\} \qquad (30)$$
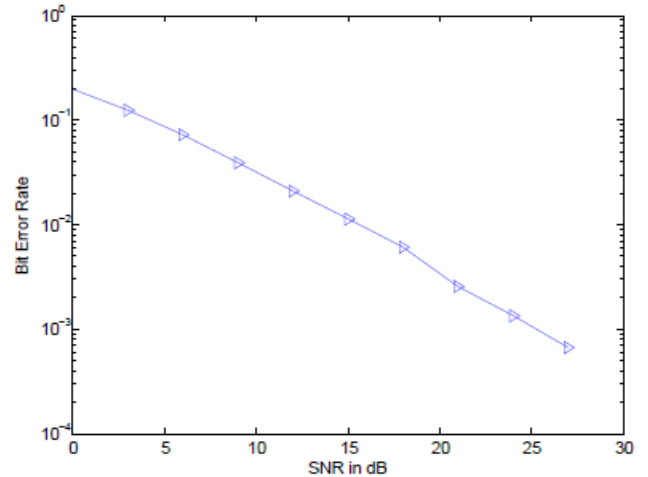
4. SIMULATION RESULTS

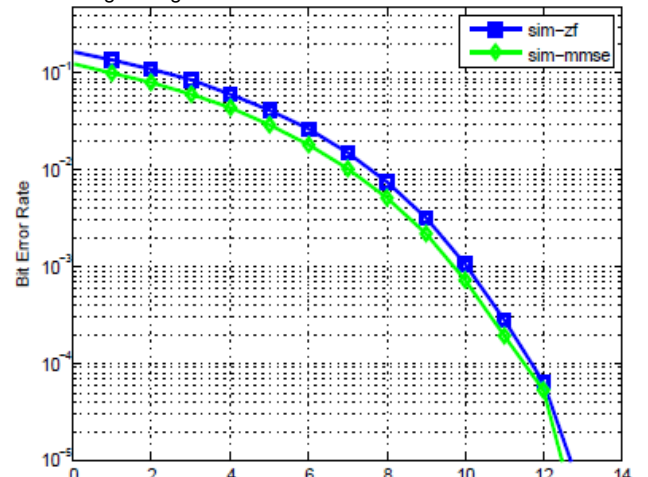Fig 4. Bit error rate for BPSK modulation technique Using LS algorithm

Fig 5. Bit error rate for BPSK modulation technique Using MMSE algorithm
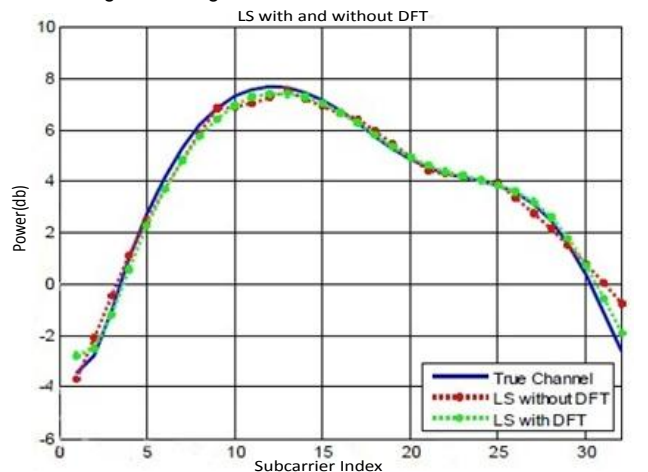

Fig 6 LS-linear channel estimation for QPSK modulation technique with DFT and without DFT [2]
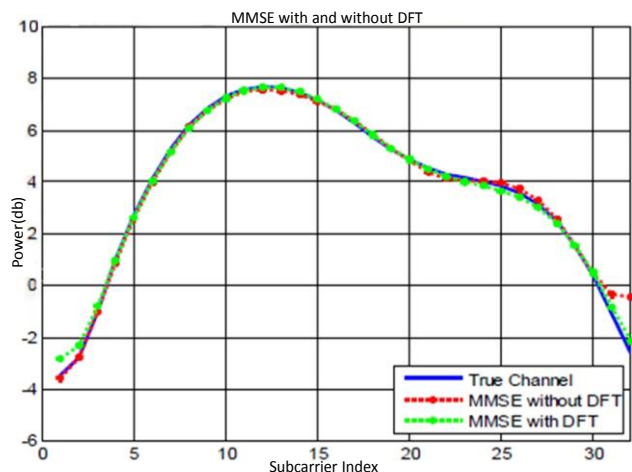
Fig 7 MMSE-linear channel estimation for QPSK modulation technique with DFT and without DFT [2]
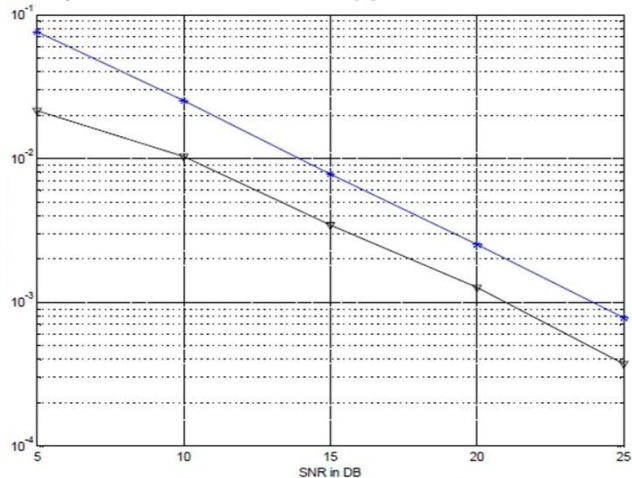

Fig 8. Plot of SNR v/s MSE for an OFDM system with MMSE\LS estimator based receivers
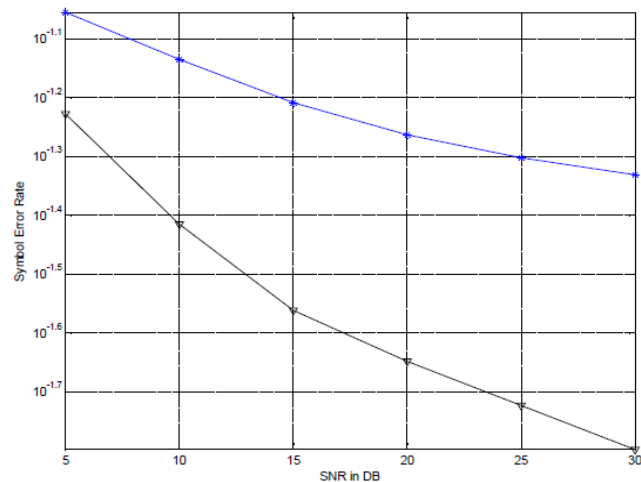

Fig 9. Plot of SNR v/s MSE for an OFDM system with MMSE|LS estimator based receivers

CONCLUSION

The combination of OFDM with Multiple Input and Multiple Output has fulfilled the future needs of high transmission rate and reliability. The quality of transmission can be further improved by reducing the effect of fading, which can be reduced by properly estimating the channel at the receiver side. For high

SNRs the LSE estimator is both simple and adequate. The MMSE estimator has good performance but high complexity. To further improve the performance of LSE and MMSE, DFT based channel estimation is applied. For subcarrier index 10, true channel power comes out to be 7.2dB. Estimated power is calculated using LS linear, and MMSE as 6.8 dB, 7.25dB and 7.20 dB and performance is improved by 0.52 dB, 0.002 dB and 0.0.02 dB respectively with application of DFT technique.Due to simplicity of LS method, however, the LS method has been widely used for channel estimation. It is clear that the MMSE estimation shows better performance than the LS estimation does at the cost of requiring the additional computation and information on the channel characteristics.

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